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Axial form-factor and induced pseudoscalar form-factor of the nucleons

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Abstract. We calculate the axial and the induced pseudoscalar form-factors $G_A(t=-Q^2)$ and $G_P(t=-Q^2)$ of the nucleons in the framework of the light-cone QCD sum-rules approach up to twist-6 three valence quark light-cone distribution amplitudes, and observe that the form-factors $G_A(t=-Q^2)$ and $G_P(t=-Q^2)$ at intermediate and large momentum transfers with $Q^2 > 2\,\text{GeV}^2$ have significant contributions from the end-point (soft) terms. The numerical values for the axial form-factor $G_A(t=-Q^2)$ are compatible with the experimental data and theoretical calculations; for example, the chiral quark models and lattice QCD. The numerical values for the induced pseudoscalar form-factor $G_P(t=-Q^2)$ are compatible with the calculation from the Bethe–Salpeter equation.

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1 Introduction

The axial and induced pseudoscalar form-factors of the nucleons are of fundamental importance in studying the weak interactions and pion–nucleon scattering. They provide an important test for theories which attempt to describe the substructures of the nucleons and the underlying dynamics [1, 2]. Using Lorentz covariance and chiral symmetry, the matrix element of the axial-vector current between the initial and final nucleon states excluding the second class current [3] can be parameterized as follows:

$$\langle N(P')|A^a_\mu(0)|N(P)\rangle = N(P')\left\{\gamma_\mu G_A(t) + \frac{(P'-P)_\mu}{2M} G_P(t)\right\}\gamma_5\tau^a N(P)\,; \quad (1)$$

here the τ^a are the Pauli matrices, M is the average mass for the proton and neutron, $t=(P'-P)^2$, the $G_A(t)$ and $G_P(t)$ are the axial and induced pseudoscalar form-factors, respectively. The Goldberger–Treiman relation [4] relates the form-factors $G_A(t)$ and $G_P(t)$, and the pion decay constant f_π,

$$g_{\pi NN} = \frac{g_A M}{f_\pi}, \quad g_{\pi NN} = G_P(t=m_\pi^2), \quad g_A = G_A(t=0)\,. \quad (2)$$

In this article, we calculate the axial form-factor $G_A(t)$ and the induced pseudoscalar form-factor $G_P(t)$ of the nucleons in the framework of the light-cone sum-rules (LCSR) approach [5, 6] which combine the standard techniques of the QCD sum rules with the conventional parton distribution amplitudes describing the hard exclusive processes [7]. In the LCSR approach, the short-distance operator product expansion with the vacuum condensates of increasing dimensions is replaced by the light-cone expansion with the distribution amplitudes (which corresponds to the sum of an infinite series of operators with the same twist) of increasing twists to parameterize the non-perturbative QCD vacuum, while the contributions from the hard re-scattering can be correctly incorporated as the $O(\alpha_\text{s})$ corrections [8]. In recent years, there have been a lot of applications of the LCSR to the mesons, for example, the form-factors, strong coupling constants and hadronic matrix elements [6]; the applications to the baryons are cumbersome and only the electromagnetic form-factors [9], the scalar form-factor [10] and the weak decay $\Lambda_b \to p\ell\nu_\ell$ [11] are studied, the higher twists distribution amplitudes of the baryons not having been available until recently [13].

The article is arranged as follows: we derive the light-cone sum rules for the axial and induced pseudoscalar form-factors $G_A(t)$ and $G_P(t)$ of the nucleons in Sect. 2; in Sect. 3, are numerical results and a discussion; Sect. 4 is reserved for our conclusions.

2 Light-cone sum rules for the form-factors $G_A(t)$ and $G_P(t)$

In the following, we write down the two-point correlation function $\Pi_\mu(P,q)$ in the framework of the LCSR approach,

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$$\Pi_\mu(P,q) = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\{\eta(0)J_\mu(x)\}|P\rangle\,, \quad (3)$$

with the axial-vector current

$$J_\mu(x) = \bar{d}(x)\gamma_\mu\gamma_5 u(x)\,, \quad (4)$$

and the neutron current [12]

$$\eta(0) = \epsilon^{ijk}\left[d^i(0)C \not{z} d^j(0)\right]\gamma_5 \not{z} u^k(0)\,,$$
$$\langle 0|\eta(0)|P\rangle = f_N\,(P\cdot z)\not{z}N(P)\,; \quad (5)$$

here the z is a light-cone vector, $z^2=0$, and the f_N is the coupling constant of the leading twist light-cone distribution amplitude [14]. At large Euclidean momenta $P'^2 = (P-q)^2$ and $q^2=-Q^2$, the correlation function $\Pi_\mu(P,q)$ can be calculated in perturbation theory. In the calculation, we need the following light-cone expanded quark propagator [15]:

$$S(x) = \frac{\mathrm{i}\Gamma(d/2)\not{x}}{2\pi^2(-x^2)^{d/2}} + \frac{\mathrm{i}\Gamma(d/2-1)}{16\pi^2(-x^2)^{d/2-1}} \times \int_0^1 \mathrm{d}v\Big\{(1-v)\not{x}\sigma_{\mu\nu}G^{\mu\nu}(vx) + v\sigma_{\mu\nu}G^{\mu\nu}(vx)\not{x}\Big\} + \ldots\,, \quad (6)$$

where $G_{\mu\nu} = g_s G^a_{\mu\nu}(\lambda^a/2)$ is the gluon field strength tensor and d is the space-time dimension. The contributions proportional to the $G_{\mu\nu}$ can give rise to four-particle (and five-particle) nucleon distribution amplitudes with a gluon (or quark–antiquark pair) in addition to the three valence quarks, but their corrections are usually not expected to play any significant roles [16] and are neglected here [9, 11]. In the parton model, at large momentum transfers, the electromagnetic and weak currents interact with the almost free partons in the nucleons. Employing the "free" light-cone quark propagator in the correlation function $\Pi_\mu(P,q)$, we obtain

$$z^\mu\Pi_\mu(P,q) = -2\int \mathrm{d}^4x\,\frac{z\cdot x\mathrm{e}^{\mathrm{i}q\cdot x}}{\pi^2x^4}(\gamma_5\not{z})^{\lambda\alpha}(C\not{z}\gamma_5)^{\beta\gamma}\epsilon^{ijk} \times\left\langle 0|T\left\{u^i_\alpha(0)u^j_\beta(x)d^k_\gamma(0)\right\}|P\right\rangle\,. \quad (7)$$

In the light-cone limit $x^2\to 0$, the remaining three-quark operator sandwiched between the proton state and the vacuum can be written in terms of the nucleon distribution amplitudes [12–14]. The three valence quark components of the nucleon distribution amplitudes are defined by the matrix element

$$\begin{aligned}
&4\left\langle 0|\epsilon^{ijk}u^i_\alpha(a_1x)u^j_\beta(a_2x)u^k_\gamma(a_3x)|P\right\rangle\\
&= \mathcal{S}_1MC_{\alpha\beta}(\gamma_5N)_\gamma + \mathcal{S}_2M^2C_{\alpha\beta}(\not{x}\gamma_5N)_\gamma\\
&\quad+\mathcal{P}_1M(\gamma_5C)_{\alpha\beta}N_\gamma + \mathcal{P}_2M^2(\gamma_5C)_{\alpha\beta}(\not{x}N)_\gamma\\
&\quad+\left(\mathcal{V}_1+\frac{x^2M^2}{4}\mathcal{V}_1^M\right)(\not{P}C)_{\alpha\beta}(\gamma_5N)_\gamma\\
&\quad+\mathcal{V}_2M(\not{P}C)_{\alpha\beta}(\not{x}\gamma_5N)_\gamma + \mathcal{V}_3M(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu\gamma_5N)_\gamma\\
&\quad+\mathcal{V}_4M^2(\not{x}C)_{\alpha\beta}(\gamma_5N)_\gamma\\
&\quad+\mathcal{V}_5M^2(\gamma_\mu C)_{\alpha\beta}(\mathrm{i}\sigma^{\mu\nu}x_\nu\gamma_5N)_\gamma\\
&\quad+\mathcal{V}_6M^3(\not{x}C)_{\alpha\beta}(\not{x}\gamma_5N)_\gamma\\
&\quad+\left(\mathcal{A}_1+\frac{x^2M^2}{4}\mathcal{A}_1^M\right)(\not{P}\gamma_5C)_{\alpha\beta}N_\gamma\\
&\quad+\mathcal{A}_2M(\not{P}\gamma_5C)_{\alpha\beta}(\not{x}N)_\gamma + \mathcal{A}_3M(\gamma_\mu\gamma_5C)_{\alpha\beta}(\gamma^\mu N)_\gamma\\
&\quad+\mathcal{A}_4M^2(\not{x}\gamma_5C)_{\alpha\beta}N_\gamma + \mathcal{A}_5M^2(\gamma_\mu\gamma_5C)_{\alpha\beta}(\mathrm{i}\sigma^{\mu\nu}x_\nu N)_\gamma\\
&\quad+\mathcal{A}_6M^3(\not{x}\gamma_5C)_{\alpha\beta}(\not{x}N)_\gamma\\
&\quad+\left(\mathcal{T}_1+\frac{x^2M^2}{4}\mathcal{T}_1^M\right)(P^\nu\mathrm{i}\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5N)_\gamma\\
&\quad+\mathcal{T}_2M(x^\mu P^\nu\mathrm{i}\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma_5N)_\gamma\\
&\quad+\mathcal{T}_3M(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\gamma_5N)_\gamma\\
&\quad+\mathcal{T}_4M(P^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\,rho}x_\rho\gamma_5N)_\gamma\\
&\quad+\mathcal{T}_5M^2(x^\nu\mathrm{i}\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5N)_\gamma\\
&\quad+\mathcal{T}_6M^2(x^\mu P^\nu\mathrm{i}\sigma_{\mu\nu}C)_{\alpha\beta}(\not{x}\gamma_5N)_\gamma\\
&\quad+\mathcal{T}_7M^2(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\not{x}\gamma_5N)_\gamma\\
&\quad+\mathcal{T}_8M^3(x^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\rho}x_\rho\gamma_5N)_\gamma\,. \qquad (8)
\end{aligned}$$

The calligraphic distribution amplitudes do not have a definite twist and can be related to the ones with definite twist by

$$\mathcal{S}_1 = S_1,\qquad 2Px\mathcal{S}_2 = S_1 - S_2,$$
$$\mathcal{P}_1 = P_1,\qquad 2Px\mathcal{P}_2 = P_1 - P_2$$

for the scalar and pseudoscalar distribution amplitudes,

$$\mathcal{V}_1 = V_1\,,\qquad 2Px\mathcal{V}_2 = V_1 - V_2 - V_3,$$
$$2\mathcal{V}_3 = V_3\,,\qquad 4Px\mathcal{V}_4 = -2V_1 + V_3 + V_4 + 2V_5\,,$$
$$4Px\mathcal{V}_5 = V_4 - V_3\,,$$
$$(2Px)^2\mathcal{V}_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6$$

for the vector distribution amplitudes,

$$\mathcal{A}_1 = A_1\,,\qquad 2Px\mathcal{A}_2 = -A_1 + A_2 - A_3\,,$$
$$2\mathcal{A}_3 = A_3\,,\qquad 4Px\mathcal{A}_4 = -2A_1 - A_3 - A_4 + 2A_5\,,$$
$$4Px\mathcal{A}_5 = A_3 - A_4$$
$$(2Px)^2\mathcal{A}_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6$$

for the axial-vector distribution amplitudes, and

$$\mathcal{T}_1 = T_1\,,\qquad 2Px\mathcal{T}_2 = T_1 + T_2 - 2T_3\,,$$
$$2\mathcal{T}_3 = T_7\,,\qquad 2Px\mathcal{T}_4 = T_1 - T_2 - 2T_7\,,$$
$$2Px\mathcal{T}_5 = -T_1 + T_5 + 2T_8\,,$$
$$(2Px)^2\mathcal{T}_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8\,,$$
$$4Px\mathcal{T}_7 = T_7 - T_8\,,$$
$$(2Px)^2\mathcal{T}_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8$$

for the tensor distribution amplitudes. The light-cone distribution amplitudes $F = V_i,\ A_i,\ T_i,\ S_i,\ P_i$ can be represented as

$$F(a_ipx)=\int \mathcal{D}x e^{-ipx\Sigma_i x_i a_i}F(x_i)\,, \tag{9}$$

with

$$\mathcal{D}x=dx_1dx_2dx_3\delta(x_1+x_2+x_3-1)\,.$$

The distribution amplitudes are scale dependent and can be expanded with the operators of increasing conformal spin; we write down the explicit expressions for the V_i, A_i, T_i, S_i and P_i up to the next-to-leading conformal spin accuracy in the appendix [11,13]; in the following, we will denote "the light-cone distribution amplitudes including the next-to-leading conformal spin" as "the P-wave approximation". The V_1, A_1 and T_1 are the leading twist-3 distribution amplitudes; the S_1, P_1, V_2, V_3, A_2, A_3, T_2, T_3 and T_7 are the twist-4 distribution amplitudes; the S_2, P_2, V_4, V_5, A_4, A_5, T_4, T_5 and T_8 are the twist-5 distribution amplitudes; while the twist-6 distribution amplitudes are the V_6, A_6 and T_6. The parameters ϕ_3^0, ϕ_6^0, ϕ_4^0, ϕ_5^0, ξ_4^0, ξ_5^0, ψ_4^0, ψ_5^0, ϕ_3^-, ϕ_3^+, ϕ_4^-, ϕ_4^+, ψ_4^-, ψ_4^+, ξ_4^-, ξ_4^+, ϕ_5^-, ϕ_5^+, ψ_5^-, ψ_5^+, ξ_5^-, ξ_5^+, ϕ_6^-, ϕ_6^+ in the light-cone distribution amplitudes V_i, A_i, T_i, S_i, P_i can be expressed in terms of eight independent matrix elements of the local operators with the parameters f_N, λ_1, λ_2, V_1^d, A_1^u, f_1^d, f_2^d and f_1^u; the three parameters f_N, λ_1 and λ_2 are related to the leading order (or S-wave) contributions of the conformal spin expansion, and the remaining five parameters V_1^d, A_1^u, f_1^d, f_2^d and f_1^u are related to the next-to-leading order (or P-wave) contributions of the conformal spin expansion; the explicit expressions are given in the appendix. For details, one may consult [13].

Taking into account the three valence quark light-cone distribution amplitudes up to twist-6 and performing the integration over the x in the coordinate space, finally we obtain the following results:

$$\begin{aligned}
&z^\mu\Pi_\mu(P,q)\\
&=\not{z}\gamma_5(Pz)^2N(P)\left\{2\int_0^1 dt_2t_2\int_0^{1-t_2}dt_1\right.\\
&\quad\times\left[\frac{V_1-A_1+2T_1}{(q-t_2P)^2}+M^2\frac{V_1^u-A_1^u+2T_1^u}{(q-t_2P)^4}\right]\\
&\quad+2M^2\int_0^1 d\lambda\lambda^2\int_1^\lambda dt_2\int_0^{1-t_2}dt_1\frac{1}{(q-\lambda P)^4}\\
&\quad\times[-V_1+V_4+V_5+A_1+A_4-A_5\\
&\quad-2T_1+2T_2-2T_3+2T_5+2T_7+4T_8]\\
&\quad-4M^2\int_0^1 d\tau\tau\int_1^\tau d\lambda\int_1^\lambda dt_2\int_0^{1-t_2}dt_1\\
&\quad\times\frac{T_1-T_3-T_4+T_5+T_7+T_8}{(q-\tau P)^4}\\
&\quad+8M^2\int_0^1 d\tau\tau\int_1^\tau d\lambda\int_1^\lambda dt_2\int_0^{1-t_2}dt_1\\
&\quad\left.\times\frac{q^2-\tau^2M^2}{(q-\tau P)^4}[T_1-T_3-T_4+T_5+T_7+T_8]\right\}\\
&\quad+\not{z}\not{q}\gamma_5(Pzqz)N(P)\\
&\quad\times\left\{2M\int_0^1 d\lambda\int_1^\lambda dt_2\int_0^{1-t_2}dt_1\frac{1}{(q-\lambda P)^4}\right.\\
&\quad\times\{V_1-V_2-V_3-A_1+A_2-A_3+2T_1-2T_3-2T_7\}\\
&\quad+8M^3\int_0^1 d\tau\tau\int_1^\tau d\lambda\int_1^\lambda dt_2\int_0^{1-t_2}dt_1\frac{1}{(q-\tau P)^6}\\
&\quad\times[-V_1+V_2+V_3+V_4+V_5\\
&\quad-V_6+A_1-A_2+A_3+A_4-A_5+A_6\\
&\quad\left.-2T_1+2T_3+2T_4-2T_6+2T_7+2T_8]\right\}+\dots\,;
\end{aligned}\tag{10}$$

here the $V_i=V_i(t_1,t_2,1-t_1-t_2)$, $A_i=A_i(t_1,t_2,1-t_1-t_2)$ and $T_i=T_i(t_1,t_2,1-t_1-t_2)$.

According to the basic assumption of current–hadron duality in the QCD sum-rules approach [7], we insert a complete series of intermediate states satisfying the unitarity principle with the same quantum numbers as the current operator $\eta(0)$ into the correlation function in (3) to obtain the hadronic representation. After isolating the pole term of the lowest neutron state, we obtain the following result:

$$\begin{aligned}
z^\mu\Pi_\mu(P,q)&=\frac{\not{z}P'zf_NN(P')\langle N(P')|\bar{d}(0)\not{z}\gamma_5u(0)|N(P)\rangle}{M^2-(q-P)^2}\\
&\quad+\dots\\
&=\frac{Pzf_N\left\{2PzG_A(t)\not{z}+\frac{qz}{2M}G_P(t)\not{z}\not{q}\right\}\gamma_5N(P)}{M^2-(q-P)^2}\\
&\quad+\dots
\end{aligned}\tag{11}$$

We choose the tensor structure $\not{z}\gamma_5(Pz)^2$ and $\not{z}\not{q}\gamma_5(Pzqz)$ to analyze the axial form-factor $G_A(t=-Q^2)$ and induced pseudoscalar form-factor $G_P(t=-Q^2)$ respectively.

The Borel transformation and the continuum states subtraction can be performed by using the following substitution rules:

$$\begin{aligned}
\int dx\frac{\rho(x)}{(q-xP)^2}&=-\int_0^1\frac{dx}{x}\frac{\rho(x)}{s-P'^2}\\
&\Rightarrow-\int_{x_0}^1\frac{dx}{x}\rho(x)e^{-\frac{s}{M_B^2}}\,,\\
\int dx\frac{\rho(x)}{(q-xP)^4}&=\int_0^1\frac{dx}{x^2}\frac{\rho(x)}{(s-P'^2)^2}\\
&\Rightarrow\frac{1}{M_B^2}\int_{x_0}^1\frac{dx}{x^2}\rho(x)e^{-\frac{s}{M_B^2}}+\frac{\rho(x_0)e^{-\frac{s_0}{M_B^2}}}{Q^2+x_0^2M^2}\,,\\
\int dx\frac{\rho(x)}{(q-xP)^6}&=-\int_0^1\frac{dx}{x^3}\frac{\rho(x)}{(s-P'^2)^3}\\
&\Rightarrow-\frac{1}{2M_B^4}\int_{x_0}^1\frac{dx}{x^3}\rho(x)e^{-\frac{s}{M_B^2}}\\
&\quad-\frac{\rho(x_0)e^{-\frac{s_0}{M_B^2}}}{2x_0(Q^2+x_0^2M^2)}+\frac{x_0^2}{2(Q^2+x_0^2M^2)}\\
&\quad\times\left[\frac{d}{dx_0}\frac{\rho(x_0)}{x_0(Q^2+x_0^2M^2)}\right]e^{-\frac{s_0}{M_B^2}}\,,
\end{aligned}$$

$$s=(1-x)M^2+\frac{(1-x)}{x}Q^2\,,$$
$$x_0=\frac{\left|\begin{array}{c}\sqrt{(Q^2+s_0-M^2)^2+4M^2Q^2}\\-(Q^2+s_0-M^2)\end{array}\right|}{2M^2}\,. \qquad (12)$$

Finally we obtain the sum rule for the axial form-factor $G_A(t=-Q^2)$ and the induced pseudoscalar form-factor $G_P(t=-Q^2)$,

$$\begin{aligned}
&G_A(t)f_N\mathrm{e}^{-\frac{M^2}{M_\mathrm{B}^2}}\\
&=-\int_{x_0}^1\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\exp\left\{-\frac{t_2(1-t_2)M^2+(1-t_2)Q^2}{t_2M_\mathrm{B}^2}\right\}\\
&\quad\times[V_1-A_1+2T_2]\\
&\quad+\frac{M^2}{M_\mathrm{B}^2}\int_{x_0}^1\frac{\mathrm{d}t_2}{t_2}\int_0^{1-t_2}\mathrm{d}t_1\\
&\quad\times\exp\left\{-\frac{t_2(1-t_2)M^2+(1-t_2)Q^2}{t_2M_\mathrm{B}^2}\right\}\left[V_1^u-A_1^u+2T_2^u\right]\\
&\quad+\frac{x_0M^2}{Q^2+x_0^2M^2}\int_0^{1-x_0}\mathrm{d}t_1\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}\left[V_1^u-A_1^u+2T_2^u\right]\\
&\quad+\frac{M^2}{M_\mathrm{B}^2}\int_{x_0}^1\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\\
&\quad\times\exp\left\{-\frac{\lambda(1-\lambda)M^2+(1-\lambda)Q^2}{\lambda M_\mathrm{B}^2}\right\}\\
&\quad\times[-V_1+V_4+V_5+A_1+A_4-A_5\\
&\quad-2T_1+2T_2-2T_3+2T_5+2T_7+4T_8]\\
&\quad+\frac{x_0^2M^2}{Q^2+x_0^2M^2}\int_1^{x_0}\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}\\
&\quad\times[-V_1+V_4+V_5+A_1+A_4-A_5\\
&\quad-2T_1+2T_2-2T_3+2T_5+2T_7+4T_8]\\
&\quad-\frac{2M^2}{M_\mathrm{B}^2}\int_{x_0}^1\frac{\tau}{\tau}\int_1^\tau\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\\
&\quad\times\exp\left\{-\frac{\tau(1-\tau)M^2+(1-\tau)Q^2}{\tau M_\mathrm{B}^2}\right\}\\
&\quad\times[T_1-T_3-T_4+T_5+T_7+T_8]\\
&\quad-\frac{2x_0M^2}{Q^2+x_0^2M^2}\int_1^{x_0}\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}\\
&\quad\times[T_1-T_3-T_4+T_5+T_7+T_8]\\
&\quad+\frac{2M^2}{M_\mathrm{B}^4}\int_{x_0}^1\frac{\tau}{\tau^2}\int_1^\tau\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\\
&\quad\times\exp\left\{-\frac{\tau(1-\tau)M^2+(1-\tau)Q^2}{\tau M_\mathrm{B}^2}\right\}\\
&\quad\times\left[Q^2+\tau^2M^2\right][T_1-T_3-T_4+T_5+T_7+T_8]\\
&\quad+\frac{2M^2}{M_\mathrm{B}^2}\int_1^{x_0}\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}\\
&\quad\times[T_1-T_3-T_4+T_5+T_7+T_8]\\
&\quad-\frac{2x_0^2M^2}{Q^2+x_0^2M^2}\int_1^{x_0}\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}\\
&\quad\times[T_1-T_3-T_4+T_5+T_7+T_8]\;; \qquad (13)
\end{aligned}$$

$$\begin{aligned}
&G_P(t)f_N\mathrm{e}^{-\frac{M^2}{M_\mathrm{B}^2}}\\
&=\frac{4M^2}{M_\mathrm{B}^2}\int_{x_0}^1\frac{\mathrm{d}\lambda}{\lambda^2}\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\\
&\quad\times\exp\left\{-\frac{\lambda(1-\lambda)M^2+(1-\lambda)Q^2}{\lambda M_\mathrm{B}^2}\right\}\\
&\quad\times[V_1-V_2-V_3-A_1+A_2-A_3+2T_1-2T_3-2T_7]\\
&\quad+\frac{4M^2}{Q^2+x_0^2M^2}\int_1^{x_0}\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}\\
&\quad\times[V_1-V_2-V_3-A_1+A_2-A_3+2T_1-2T_3-2T_7]\\
&\quad-\frac{8M^4}{M_\mathrm{B}^4}\int_{x_0}^1\frac{\tau}{\tau^2}\int_1^\tau\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\\
&\quad\times\exp\left\{-\frac{\tau(1-\tau)M^2+(1-\tau)Q^2}{\tau M_\mathrm{B}^2}\right\}\\
&\quad\times[-V_1+V_2+V_3+V_4+V_5-V_6\\
&\quad+A_1-A_2+A_3+A_4-A_5+A_6\\
&\quad-2T_1+2T_3+2T_4-2T_6+2T_7+2T_8]\\
&\quad-\frac{8M^4}{(Q^2+x_0^2M^2)M_\mathrm{B}^2}\int_1^{x_0}\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\\
&\quad\times\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}[-V_1+V_2+V_3+V_4+V_5\\
&\quad-V_6+A_1-A_2+A_3+A_4-A_5+A_6\\
&\quad-2T_1+2T_3+2T_4-2T_6+2T_7+2T_8]\\
&\quad-\frac{8x_0^2M^4}{Q^2+x_0^2M^2}\left[\frac{\mathrm{d}}{\mathrm{d}x_0}\frac{1}{Q^2+x_0^2}\int_1^{x_0}\mathrm{d}\lambda\int_1^\lambda\mathrm{d}t_2\int_0^{1-t_2}\mathrm{d}t_1\right]\\
&\quad\times\exp\left\{-\frac{s_0}{M_\mathrm{B}^2}\right\}[-V_1+V_2+V_3\\
&\quad+V_4+V_5-V_6+A_1-A_2+A_3+A_4-A_5+A_6\\
&\quad-2T_1+2T_3+2T_4-2T_6+2T_7+2T_8]\,. \qquad (14)
\end{aligned}$$

3 Numerical results and discussion

The input parameters have to be specified before the numerical analysis. We choose a range for the Borel parameter that is suitable: M_B, $1.5\,\mathrm{GeV}^2<M_\mathrm{B}^2<2.5\,\mathrm{GeV}^2$. In this range, the Borel parameter M_B is small enough to warrant the higher mass resonances, and continuum states are suppressed sufficiently; on the other hand, it is large enough to warrant the convergence of the light-cone expansion with increasing twists in the perturbative QCD calculation [17, 18]. The numerical results indicate that in this range the form-factors $G_A(t=-Q^2)$ and $G_P(t=-Q^2)$ are almost independent on the Borel parameter M_B, which we can see from Figs. 1 and 2 respectively for the central values of the eight input parameters f_N, λ_1, λ_2, V_1^d, A_1^u, f_d^1, f_d^2 and f_u^1.

For simplicity, we choose the standard values for the threshold parameter s_0, $s_0=2.25\,\mathrm{GeV}^2$, to subtract the contributions from the higher resonances and continuum states i.e. we restrict the range of the integral to the energy region below the Roper resonance ($N(1440)$); furthermore, it is large enough to take into account all contributions from the neutron. For $Q^2 = (2$–$9\,\mathrm{GeV}^2$, $x \geq x_0 = 0.5$–0.8, the average value $\langle x \rangle = 0.75$–0.90, with the intermediate and large space-like momentum Q^2, the end-point (soft) contributions (or the Feynman mechanism) are dominant; it is consistent with the growing consensus that the onset of the perturbative QCD region in exclusive processes is postponed to very large energy scales. The parameters in the light-cone distribution amplitudes ϕ_3^0, ϕ_6^0, ϕ_4^0, ϕ_5^0, ξ_4^0, ξ_5^0, ψ_4^0, ψ_5^0, ϕ_3^-, ϕ_3^+, ϕ_4^-, ϕ_4^+, ψ_4^-, ψ_4^+, ξ_4^-, ξ_4^+, ϕ_5^-, ϕ_5^+, ψ_5^-, ψ_5^+, ξ_5^- ,ξ_5^+, ϕ_6^-, ϕ_6^+ are scale dependent and can be calculated with the corresponding QCD sum rules. They are functions of the eight independent parameters f_N, λ_1, λ_2, V_1^d, A_1^u, f_1^d, f_2^d and f_1^u, and the three parameters f_N, λ_1 and λ_2 are related to the leading order (or S-wave) contributions in the conformal spin expansion, the remaining five parameters V_1^d, A_1^u, f_1^d, f_2^d and f_1^u are related to the next-to-leading order (or P-wave) contributions in the conformal spin expansion; the explicit expressions are presented in the appendix, for detailed and systematic studies about this subject, one can consult [13].

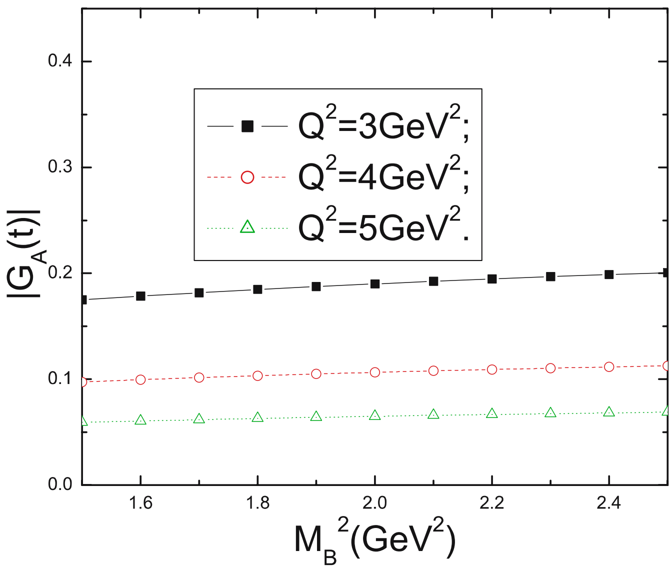

Fig. 1. The axial form-factor $G_A(t)$ with the parameter M_B for $s_0 = 2.25\,\mathrm{GeV}^2$

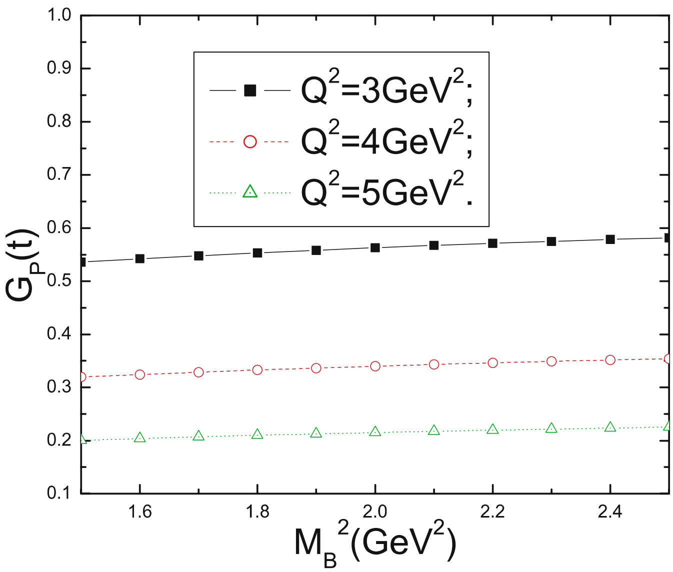

Fig. 2. The induced pseudoscalar form-factor $G_P(t)$ with the parameter M_B for $s_0 = 2.25\,\mathrm{GeV}^2$

Here we take the values at the energy scale $\mu = 1\,\mathrm{GeV}$ and neglect the evolution with the energy scale μ for simplicity; the values for the eight independent parameters are taken as $f_N = (5.3 \pm 0.5 \times 10^{-3}\,\mathrm{GeV}^2$, $\lambda_1 = -(2.7 \pm 0.9 \times 10^{-2}\,\mathrm{GeV}^2$, $\lambda_2 = (5.1 \pm 1.9 \times 10^{-2}\,\mathrm{GeV}^2$, $V_1^d = 0.23 \pm 0.03$, $A_1^u = 0.38 \pm 0.15$, $f_1^d = 0.6 \pm 0.2$, $f_2^d = 0.15 \pm 0.06$ and $f_1^u = 0.22 \pm 0.15$. In estimating those coefficients with the QCD sum rules, only the first few moments are taken into account; the values are not very accurate. In the limit $Q^2 \to \infty$, the five parameters related to the light-cone distribution amplitudes with the P-wave conformal spin take the asymptotic values $f_1^d = \frac{3}{10}$, $f_2^d = \frac{4}{15}$, $f_1^u = \frac{1}{10}$, $A_1^u = 0$ and $V_1^d = \frac{1}{3}$.

We perform the operator product expansion in the light-cone with large Q^2 and P'^2, and the form-factors $G_A(t = -Q^2)$ and $G_P(t = -Q^2)$ make sense at the regions, for example, $Q^2 > 2\,\mathrm{GeV}^2$, but with low momentum transfers, the operator product expansion is questionable. In numerical analysis we observe that the axial form-factor $G_A(t = -Q^2)$ is sensitive to the two parameters λ_1 and f_1^d, and small variations of the two parameters can lead to relatively large changes for the values. The induced pseudoscalar form-factor $G_P(t = -Q^2)$ is sensitive to the four parameters f_N, λ_1, f_1^d and f_1^u; small variations of those parameters, especially λ_1 and f_1^d, can lead to large changes for the values, which are shown in Figs. 3–8, respectively. The large uncertainties can impair the the predictive ability of the sum rules. The parameters λ_1, f_1^d, f_N and f_1^u should be refined to make robust predictions, and in [10], we observe that the scalar form-factor of the nucleon is sensitive to the four parameters λ_1, f_1^d, f_2^d and f_1^u, so refining the three parameters λ_1, f_1^d and f_1^u is of great importance. The final numerical values for the axial form-factor $G_A(t = -Q^2)$ and the induced pseudoscalar form-factor $G_P(t = -Q^2)$ at the intermediate and large space-like momentum regions, $2\,\mathrm{GeV}^2 < Q^2 < 9\,\mathrm{GeV}^2$, are plotted in Figs. 9 and 10 respectively.

From those figures, we can see that the central values of the axial form-factor $G_A(t = -Q^2)$ lie above the results of the double-pole fitted formulation from the neutrino scattering experiments [2],

$$G_A(t) = \frac{g_A}{(1 - t/M_A^2)^2}\,, \tag{15}$$

where we take the values $g_A = 1.2673$, $M_A = 1.026$, and neglect the uncertainties for simplicity; at the region $Q^2 > 4.0\,\mathrm{GeV}^2$, the values of the double-pole fitted formu-

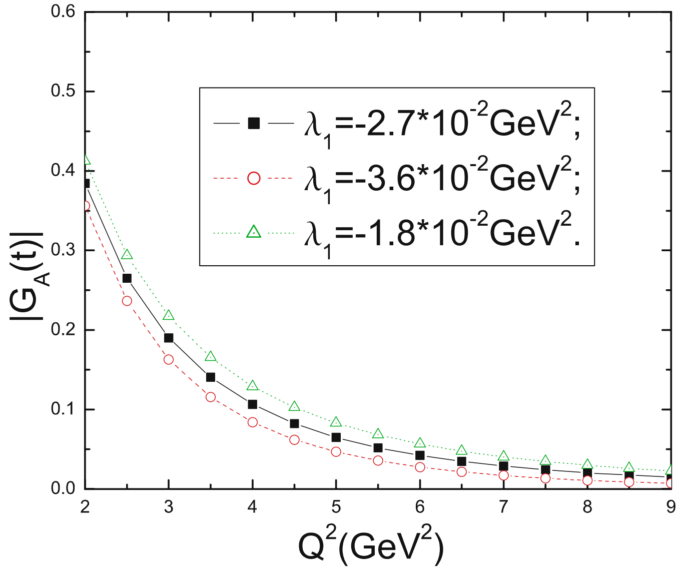

Fig. 3. The axial form-factor $G_A(t)$ with the parameters $M_{\mathrm{B}}^2 = 2.0\ \mathrm{GeV}^2$ and λ_1

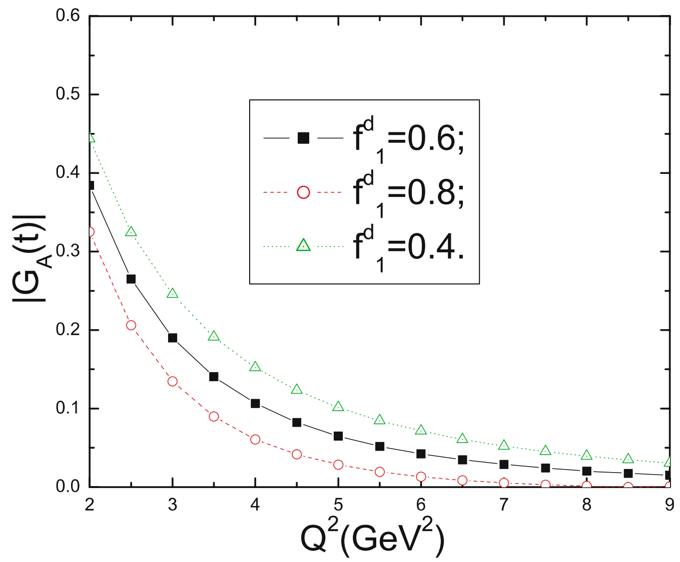

Fig. 4. The axial form-factor $G_A(t)$ with the parameters $M_{\mathrm{B}}^2 = 2.0\ \mathrm{GeV}^2$ and f_1^d

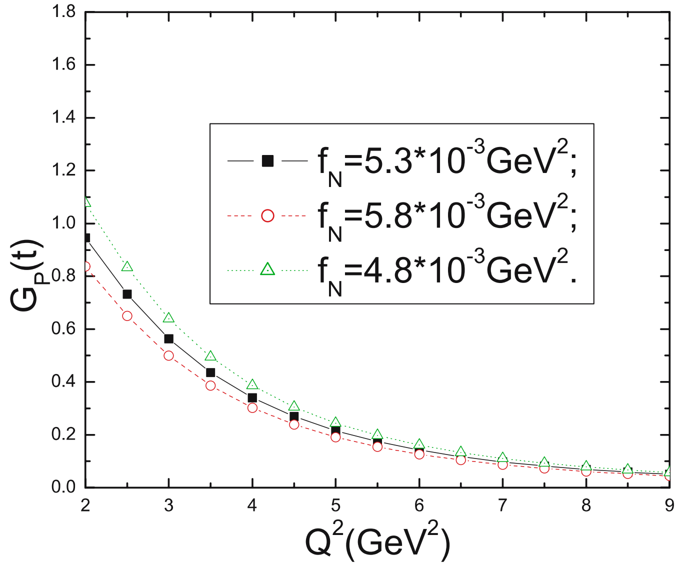

Fig. 5. The induced pseudoscalar form-factor $G_P(t)$ with the parameters $M_{\mathrm{B}}^2 = 2.0\ \mathrm{GeV}^2$ and f_N

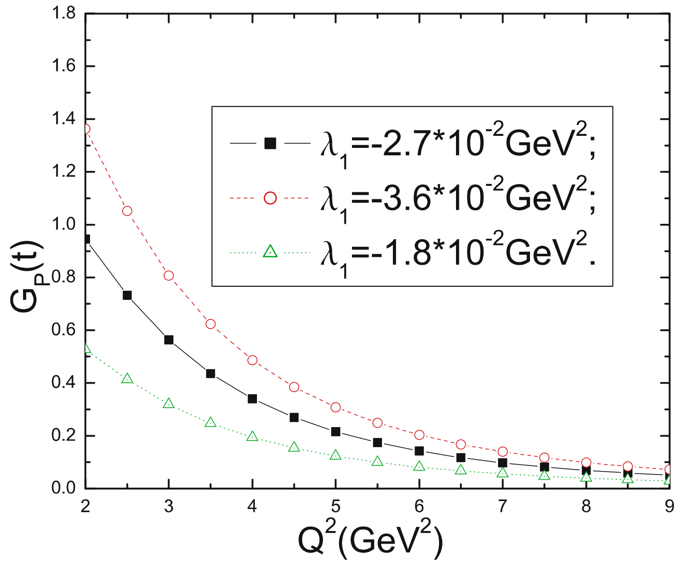

Fig. 6. The induced pseudoscalar form-factor $G_P(t)$ with the parameters $M_{\mathrm{B}}^2 = 2.0\ \mathrm{GeV}^2$ and λ_1

lation lie between the up and down limits, and our results can make both qualitative and quantitative predictions. Furthermore, our results are compatible with the calculations of lattice QCD [19] and chiral quark models [20]. For the induced pseudoscalar form-factor $G_P(t = -Q^2)$, the uncertainties are very large and the values make sense only qualitatively, not quantitatively; our results are compatible with the calculation from the Bethe–Salpeter equation [21].

In the limit $Q^2 \to \infty$, we present the numerical values for the axial form-factor $G_A(t = -Q^2)$ and the induced pseudoscalar form-factor $G_P(t = -Q^2)$ with the asymptotic light-cone distribution amplitudes in Figs. 11 and 12, respectively. From Fig. 11, we can see that for the axial form-factor $G_A(t = -Q^2)$ the values with the asymptotic light-cone distribution amplitudes lie above the corresponding ones with the light-cone distribution amplitudes in the P-wave approximation, and at $Q^2 > 10\,\mathrm{GeV}^2$ the two curves approach the values of the double-pole fitted formulation $G_A(t = -Q^2) \sim \frac{1}{Q^4}$, which is expected from the naive power counting rules. From Fig. 12, we can see that for the induced pseudoscalar form-factor $G_P(t = -Q^2)$, the values with the asymptotic light-cone distribution amplitudes have a negative sign compared to the corresponding ones with the light-cone distribution ampli-

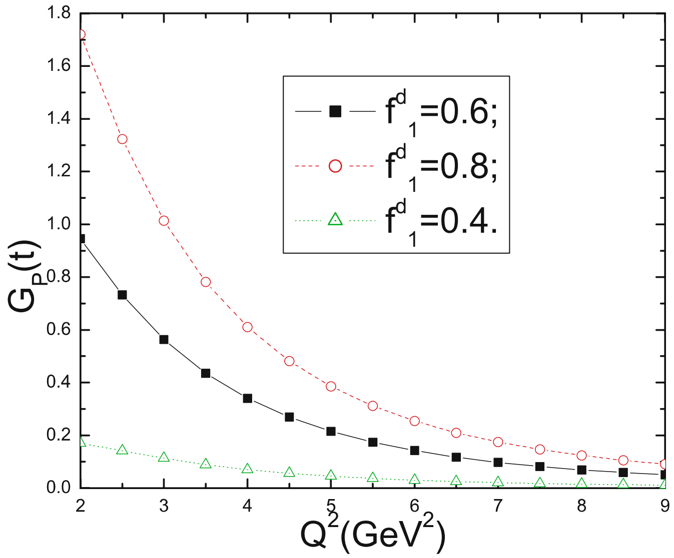

Fig. 7. The induced pseudoscalar form-factor $G_P(t)$ with the parameters $M_{\mathrm{B}}^2 = 2.0\,\mathrm{GeV}^2$ and f_1^d

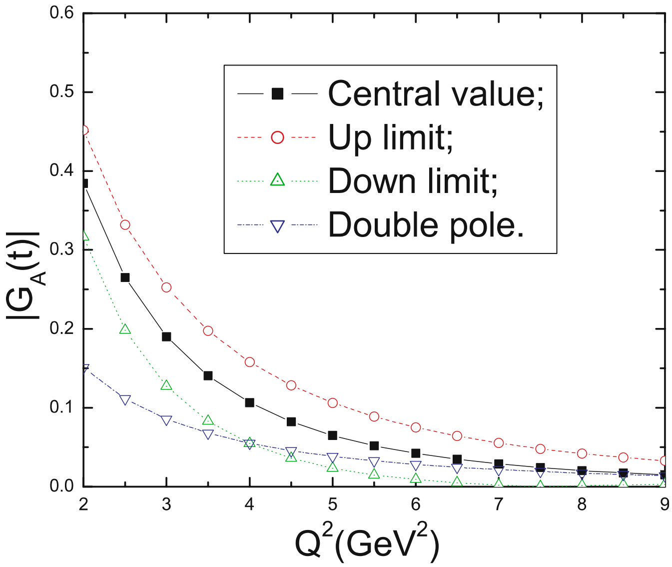

Fig. 9. The axial form-factor $G_A(t)$ with the parameter $M_{\mathrm{B}}^2 = 2.0\,\mathrm{GeV}^2$

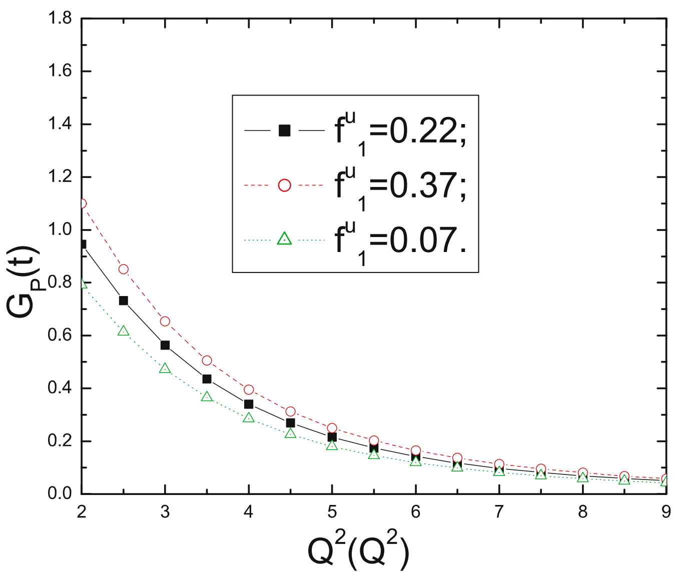

Fig. 8. The induced pseudoscalar form-factor $G_P(t)$ with the parameters $M_{\mathrm{B}}^2 = 2.0\,\mathrm{GeV}^2$ and f_1^u

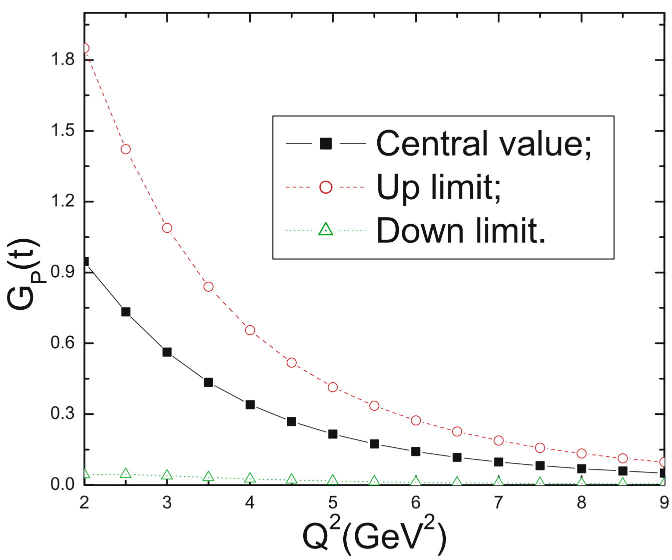

Fig. 10. The induced pseudoscalar form-factor $G_P(t)$ with the parameter $M_{\mathrm{B}}^2 = 2.0\,\mathrm{GeV}^2$

tudes in the P-wave approximation, and at $Q^2 > 10\,\mathrm{GeV}^2$ the two curves approach the same values. The large difference between the values from the asymptotic light-cone distribution amplitudes and the P-wave approximated light-cone distribution amplitudes again indicate the importance of the contributions from the P-wave conformal spin at the intermediate and large momentum transfers $2\,\mathrm{GeV}^2 < Q^2 < 10\,\mathrm{GeV}^2$; to make robust predictions, we have to refine the five parameters.

The consistent and complete LCSR analysis should take into account the contributions from the perturbative α_s corrections, the distribution amplitudes with additional valence gluons and quark–antiquark pairs, and improve the parameters which enter in the LCSRs.

4 Conclusion

In this work, we calculate the axial and induced pseudoscalar form-factors $G_A(t = -Q^2)$ and $G_P(t = -Q^2)$ of the nucleons in the framework of the LCSR approach up to twist-6 three valence quark light-cone distribution amplitudes. The form-factors $G_A(t = -Q^2)$ and $G_P(t = -Q^2)$ at intermediate and large momentum transfers with $Q^2 > 2\,\mathrm{GeV}^2$ have significant contributions from the end-point (soft) terms. The axial form-factor $G_A(t = -Q^2)$ is sensitive to the two parameters λ_1 and f_1^d, and small variations of the two parameters can lead to relatively large changes for the values; the induced pseudoscalar form-factor $G_P(t = -Q^2)$ is sensitive to the four parameters f_N,

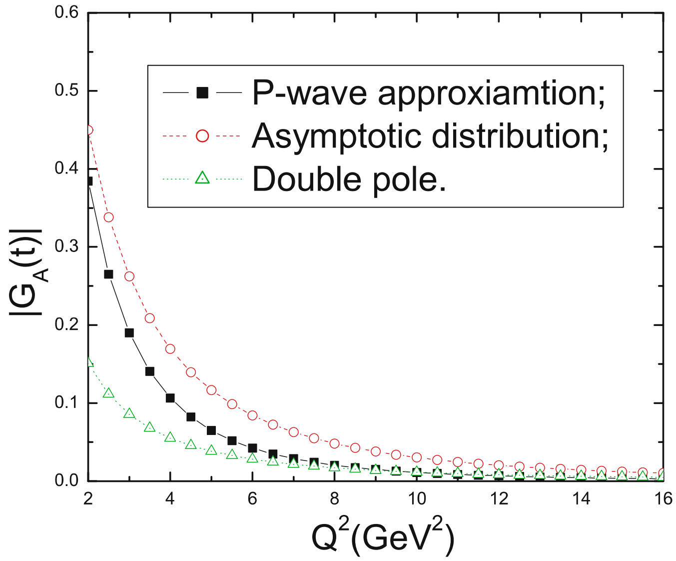

Fig. 11. The axial form-factor $G_A(t)$ with the parameters $f_N = 5.3\times10^{-3}$ GeV^2, $\lambda_1 = -2.7\times10^{-2}$ GeV^2, $\lambda_2 = 5.1\times10^{-2}$ GeV^2 and $M_B^2 = 2.0$ GeV^2

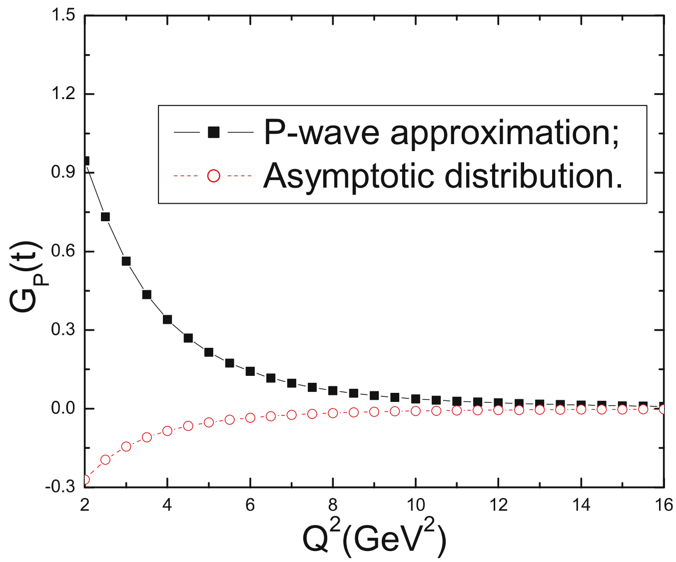

Fig. 12. The $G_P(t)$ with the parameters $f_N = 5.3\times10^{-3}$ GeV^2, $\lambda_1 = -2.7\times10^{-2}$ GeV^2, $\lambda_2 = 5.1\times10^{-2}$ GeV^2 and $M_B^2 = 2.0$ GeV^2

λ_1, f_1^d and f_1^u, and small variations of those parameters, especially λ_1 and f_1^d, can lead to large changes for the values. The large uncertainties can impair the predictive ability of the sum rules; the parameters λ_1, f_1^d, f_N and f_1^u should be refined to make robust predictions. The numerical values for the $G_A(t=-Q^2)$ are compatible with the experimental data and theoretical calculations, for example, the chiral quark model and lattice QCD. In the limit $Q^2\to\infty$, the values for the axial form-factor $G_A(t=-Q^2)$ with both the asymptotic light-cone distribution amplitudes and the light-cone distribution amplitudes in the P-wave approximation approach the results of the double-pole fitted formulation $G_A(t=-Q^2)\sim\frac{1}{Q^4}$. The numerical results for the induced pseudoscalar form-factor $G_P(t=-Q^2)$ are compatible with the calculation from the Bethe–Salpeter equation; in the limit $Q^2\to\infty$, the values of the $G_A(t=-Q^2)$ with both the asymptotic light-cone distribution amplitudes and the light-cone distribution amplitudes in the P-wave approximation approach the same values. The consistent and complete LCSR analysis should take into account the contributions from the perturbative α_s corrections, the distribution amplitudes with additional valence gluons and quark–antiquark pairs, and improve the parameters which enter in the LCSRs.

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Appendix

We have

$$\begin{aligned}
V_1(x_i,\mu) &= 120x_1x_2x_3\left[\phi_3^0(\mu)+\phi_3^+(\mu)(1-3x_3)\right],\\
V_2(x_i,\mu) &= 24x_1x_2\left[\phi_4^0(\mu)+\phi_3^+(\mu)(1-5x_3)\right],\\
V_3(x_i,\mu) &= 12x_3\Big\{\psi_4^0(\mu)(1-x_3)+\psi_4^-(\mu)\\
&\qquad\times\left[x_1^2+x_2^2-x_3(1-x_3)\right]\\
&\qquad+\psi_4^+(\mu)(1-x_3-10x_1x_2)\Big\},\\
V_4(x_i,\mu) &= 3\Big\{\psi_5^0(\mu)(1-x_3)\\
&\qquad+\psi_5^-(\mu)\left[2x_1x_2-x_3(1-x_3)\right]\\
&\qquad+\psi_5^+(\mu)\left[1-x_3-2\left(x_1^2+x_2^2\right)\right]\Big\},\\
V_5(x_i,\mu) &= 6x_3\left[\phi_5^0(\mu)+\phi_5^+(\mu)(1-2x_3)\right],\\
V_6(x_i,\mu) &= 2\left[\phi_6^0(\mu)+\phi_6^+(\mu)(1-3x_3)\right];
\end{aligned}$$

$$\begin{aligned}
A_1(x_i,\mu) &= 120x_1x_2x_3\phi_3^-(\mu)(x_2-x_1),\\
A_2(x_i,\mu) &= 24x_1x_2\phi_4^-(\mu)(x_2-x_1),\\
A_3(x_i,\mu) &= 12x_3(x_2-x_1)\Big\{\left(\psi_4^0(\mu)+\psi_4^+(\mu)\right)\\
&\qquad+\psi_4^-(\mu)(1-2x_3)\Big\},\\
A_4(x_i,\mu) &= 3(x_2-x_1)\\
&\qquad\times\Big\{-\psi_5^0(\mu)+\psi_5^-(\mu)x_3+\psi_5^+(\mu)(1-2x_3)\Big\},\\
A_5(x_i,\mu) &= 6x_3(x_2-x_1)\phi_5^-(\mu),\\
A_6(x_i,\mu) &= 2(x_2-x_1)\phi_6^-(\mu);
\end{aligned}$$

$$
\begin{aligned}
T_1(x_i,\mu) &= 120x_1x_2x_3\left[\phi_3^0(\mu)+\frac{1}{2}\left(\phi_3^- - \phi_3^+\right)\right.\\
&\quad\left.\times(\mu)(1-3x_3)\right],\\
T_2(x_i,\mu) &= 24x_1x_2\left[\xi_4^0(\mu)+\xi_4^+(\mu)(1-5x_3)\right],\\
T_3(x_i,\mu) &= 6x_3\Big\{\left(\xi_4^0+\phi_4^0+\psi_4^0\right)(\mu)(1-x_3)\\
&\quad+\left(\xi_4^- +\phi_4^- -\psi_4^-\right)(\mu)\left[x_1^2+x_2^2-x_3(1-x_3)\right]\\
&\quad+\left(\xi_4^+ +\phi_4^+ +\psi_4^+\right)(\mu)(1-x_3-10x_1x_2)\Big\},\\
T_4(x_i,\mu) &= \frac{3}{2}\Big\{\left(\xi_5^0+\phi_5^0+\psi_5^0\right)(\mu)(1-x_3)\\
&\quad+\left(\xi_5^- +\phi_5^- -\psi_5^-\right)(\mu)\left[2x_1x_2-x_3(1-x_3)\right]\\
&\quad+\left(\xi_5^+ +\phi_5^+ +\psi_5^+\right)(\mu)\left(1-x_3-2\left(x_1^2+x_2^2\right)\right)\Big\},\\
T_5(x_i,\mu) &= 6x_3\left[\xi_5^0(\mu)+\xi_5^+(\mu)(1-2x_3)\right],\\
T_6(x_i,\mu) &= 2\left[\phi_6^0(\mu)+\frac{1}{2}\left(\phi_6^- -\phi_6^+\right)(\mu)(1-3x_3)\right],\\
T_7(x_i,\mu) &= 6x_3\Big\{\left(-\xi_4^0+\phi_4^0+\psi_4^0\right)(\mu)(1-x_3)\\
&\quad+\left(-\xi_4^- +\phi_4^- -\psi_4^-\right)(\mu)\left[x_1^2+x_2^2-x_3(1-x_3)\right]\\
&\quad+\left(-\xi_4^+ +\phi_4^+ +\psi_4^+\right)(\mu)(1-x_3-10x_1x_2)\Big\},\\
T_8(x_i,\mu) &= \frac{3}{2}\Big\{\left(-\xi_5^0+\phi_5^0+\psi_5^0\right)(\mu)(1-x_3)\\
&\quad+\left(-\xi_5^- +\phi_5^- -\psi_5^-\right)(\mu)\left[2x_1x_2-x_3(1-x_3)\right]\\
&\quad+\left(-\xi_5^+ +\phi_5^+ +\psi_5^+\right)(\mu)\\
&\quad\times\left(1-x_3-2\left(x_1^2+x_2^2\right)\right)\Big\};
\end{aligned}
$$

$$
\begin{aligned}
S_1(x_i,\mu) &= 6x_3(x_2-x_1)\\
&\quad\times\left[\left(\xi_4^0+\phi_4^0+\psi_4^0+\xi_4^+ +\phi_4^+ +\psi_4^+\right)(\mu)\right.\\
&\quad\left.+\left(\xi_4^- +\phi_4^- -\psi_4^-\right)(\mu)(1-2x_3)\right],\\
S_2(x_i,\mu) &= \frac{3}{2}(x_2-x_1)\left[-\left(\psi_5^0+\phi_5^0+\xi_5^0\right)(\mu)\right.\\
&\quad+\left(\xi_5^- +\phi_5^- -\psi_5^0\right)(\mu)x_3\\
&\quad\left.+\left(\xi_5^+ +\phi_5^+ +\psi_5^0\right)(\mu)(1-2x_3)\right],\\
P_1(x_i,\mu) &= 6x_3(x_2-x_1)\left[\left(\xi_4^0-\phi_4^0-\psi_4^0+\xi_4^+ -\phi_4^+ -\psi_4^+\right)\right.\\
&\quad\left.\times(\mu)+\left(\xi_4^- -\phi_4^- +\psi_4^-\right)(\mu)(1-2x_3)\right],\\
P_2(x_i,\mu) &= \frac{3}{2}(x_2-x_1)\left[\left(\psi_5^0+\psi_5^0-\xi_5^0\right)(\mu)\right.\\
&\quad+\left(\xi_5^- -\phi_5^- +\psi_5^0\right)(\mu)x_3\\
&\quad\left.+\left(\xi_5^+ -\phi_5^+ -\psi_5^0\right)(\mu)(1-2x_3)\right];
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}_1^d(x_3) &= \frac{x_3^2}{24}\left(\lambda_1C_\lambda^d+f_NC_f^d\right),\\
\mathcal{V}_1^u(x_2) &= \frac{x_2^2}{24}\left(\lambda_1C_\lambda^u+f_NC_f^u\right),\\
C_\lambda^d &= -(1-x_3)\left[11+131\,x_3-169x_3^2+63x_3^3\right.\\
&\quad\left.-30\,f_1^d\left(3+11x_3-17x_3^2+7x_3^3\right)\right]\\
&\quad-12\left(3-10\,f_1^d\right)\ln x_3,\\
C_f^d &= -(1-x_3)\left[1441+505x_3-3371x_3^2+3405x_3^3\right.\\
&\quad-1104x_3^4\\
&\quad\left.-24V_1^d\left(207-3x_3-368x_3^2+412x_3^3-138x_3^4\right)\right]\\
&\quad-12\left(73-220\,V_1^d\right)\ln x_3,\\
C_\lambda^u &= -(1-x_2)^3\left[13-20f_1^d+3x_2+10f_1^u(1-3x_2)\right],\\
C_f^u &= (1-x_2)^3\left[113+495x_2-552x_2^2\right.\\
&\quad\left.+10A_1^u(-1+3x_2)+2V_1^d\left(113-951x_2+828x_2^2\right)\right];
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_1^d(x_3) &= 0,\\
\mathcal{A}_1^u(x_2) &= \frac{x_2^2}{24}(1-x_2)^3\left(\lambda_1D_\lambda^u+f_ND_f^u\right),\\
D_\lambda^u &= 29-45x_2-10f_1^u(7-9x_2)-20f_1^d(5-6x_2),\\
D_f^u &= 11+45x_2+10V_1^d(1-30x_2)\\
&\quad-2A_1^u\left(113-951x_2+828x_2^2\right);
\end{aligned}
$$

$$
\phi_3^0=\phi_6^0=f_N,\qquad \phi_4^0=\phi_5^0=\frac{1}{2}(\lambda_1+f_N),
$$
$$
\xi_4^0=\xi_5^0=\frac{1}{6}\lambda_2,\qquad \psi_4^0=\psi_5^0=\frac{1}{2}(f_N-\lambda_1);
$$

$$
\begin{aligned}
\tilde{\phi}_3^- &= \frac{21}{2}A_1^u,\\
\tilde{\phi}_3^+ &= \frac{7}{2}\left(1-3V_1^d\right),\\
\phi_4^- &= \frac{5}{4}\left(\lambda_1\left(1-2f_1^d-4f_1^u\right)+f_N\left(2A_1^u-1\right)\right),\\
\phi_4^+ &= \frac{1}{4}\left(\lambda_1\left(3-10f_1^d\right)-f_N\left(10V_1^d-3\right)\right),\\
\psi_4^- &= -\frac{5}{4}\left(\lambda_1\left(2-7f_1^d+f_1^u\right)+f_N\left(A_1^u+3V_1^d-2\right)\right),\\
\psi_4^+ &= -\frac{1}{4}\left(\lambda_1\left(-2+5f_1^d+5f_1^u\right)+f_N\left(2+5A_1^u-5V_1^d\right)\right),\\
\xi_4^- &= \frac{5}{16}\lambda_2\left(4-15f_2^d\right),\\
\xi_4^+ &= \frac{1}{16}\lambda_2\left(4-15f_2^d\right);
\end{aligned}
$$

$$
\begin{aligned}
\phi_5^- &= \frac{5}{3}\left(\lambda_1\left(f_1^d-f_1^u\right)+f_N\left(2A_1^u-1\right)\right),\\
\phi_5^+ &= -\frac{5}{6}\left(\lambda_1\left(4f_1^d-1\right)+f_N\left(3+4V_1^d\right)\right),\\
\psi_5^- &= \frac{5}{3}\left(\lambda_1\left(f_1^d-f_1^u\right)+f_N\left(2-A_1^u-3V_1^d\right)\right),\\
\psi_5^+ &= -\frac{5}{6}\left(\lambda_1\left(-1+2f_1^d+2f_1^u\right)+f_N\left(5+2A_1^u-2V_1^d\right)\right),\\
\xi_5^- &= -\frac{5}{4}\lambda_2f_2^d,\\
\xi_5^+ &= \frac{5}{36}\lambda_2\left(2-9f_2^d\right),
\end{aligned}
$$

$$\phi_6^- = \frac{1}{2}\left(\lambda_1\left(1-4f_1^d-2f_1^u\right)+f_N\left(1+4A_1^u\right)\right) ,$$
$$\phi_6^+ = -\frac{1}{2}\left(\lambda_1\left(1-2f_1^d\right)+f_N\left(4V_1^d-1\right)\right) .$$

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